Honors Precalculus
Test 2D Vectors

No Calculator may be used on this section.

1. The scalar \( k \) is equal to 4. Vectors \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \) are defined as \( \mathbf{v}_1 = \langle -3, 5 \rangle \) and \( \mathbf{v}_2 = \langle 2, -1 \rangle \).

   Evaluate \( k \mathbf{v}_1 + \mathbf{v}_2 \). Then find the magnitude and direction of \( k \mathbf{v}_1 + \mathbf{v}_2 \).

   \[
   4 \langle -3, 5 \rangle + \langle 2, -1 \rangle = \langle -12, 20 \rangle + \langle 2, -1 \rangle = \langle -10, 19 \rangle
   \]

   \[
   \text{Mag} = \sqrt{(-10)^2 + 19^2}
   \]

   1. \( \sqrt{419} \) \( ^\circ \) \( \tan^{-1} \left( \frac{19}{10} \right) \)

   bearing \( 270 + \tan^{-1} \left( \frac{19}{10} \right) \)

2. Find the unit vector in the direction of \( (7, -24) \).

3. Let \( \mathbf{u} = \langle -1, -1 \rangle \). Find the vector \( \mathbf{v} \) such that \( \mathbf{u} \cdot \mathbf{v} = -6 \) and \( |\mathbf{v}| = \sqrt{18} \).

   \[
   \mathbf{v} = \langle x, y \rangle
   \]

   \[
   -x + y = -6
   \]

   \[
   \sqrt{x^2 + y^2} = \sqrt{18}
   \]

   \[
   \mathbf{v} = \langle -3, 3 \rangle
   \]

4. Which of the following vectors are orthogonal?

A. \( \langle 1, 2 \rangle \) and \( \langle 2, 4 \rangle \)
B. \( \langle 1, 2 \rangle \) and \( \langle -1, -2 \rangle \)
C. \( \langle -1, -2 \rangle \) and \( \langle 1, -1 \rangle \)
D. \( \langle -1, -2 \rangle \) and \( \langle -2, 1 \rangle \)

   \[
   2 + (-2) = 0
   \]

   4. D
5. Find the distance to the line $3x - 2y + 12 = 0$ from the point $(2, -2)$.

$$d = \frac{|3(2) - 2(-2) + 12|}{\sqrt{9 + 4}} = \frac{|6 + 4 + 12|}{\sqrt{13}} = \frac{22}{\sqrt{13}}$$

6. Use the line $3x - 4y + 12 = 0$ to answer the following questions.

a. Sketch the line and positive unit normal. 

$$3x - 4y = -12$$

$$\left< -\frac{3}{5}, \frac{4}{5} \right>$$

b. Find the positive unit normal.

$$\frac{12}{5}$$

c. Find the distance from $(0,0)$ to the line.

$$\frac{12}{5}$$

d. Find the “footer” where the positive unit normal “hits” the line.

$$\left( -\frac{36}{25}, \frac{48}{25} \right)$$
A Calculator may be used on this section.

7. Find the angle between \( \mathbf{u} \) and \( \mathbf{v} \) if \( \mathbf{u} = -2\mathbf{i} + 3\mathbf{j} \) and \( \mathbf{v} = -5\mathbf{i} - \mathbf{j} \).

\[
\cos \theta = \frac{-10}{\sqrt{13} \cdot \sqrt{26}}
\]

\[
\cos \theta = \frac{3}{\sqrt{13} \cdot \sqrt{26}}
\]

\[
\theta = 67.620^\circ
\]

7. \( 67.620^\circ \)

8. Find the components of the vector \( \mathbf{v} \) with direction angle \( 242^\circ \) and length 5.

\[
\mathbf{v} = 5 \langle \cos 242^\circ, \sin 242^\circ \rangle
\]

\[
\mathbf{v} = \langle -2.347, -4.415 \rangle
\]

8. \( \langle -2.347, -4.415 \rangle \)
9. Find the length of the projection of \( \mathbf{u} = (-2, 5) \) onto \( \mathbf{v} = (3, 1) \). Sketch \( \mathbf{u} \), \( \mathbf{v} \) and the projection.

\[
\hat{\mathbf{v}} = \left< \frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}} \right>
\]

\[
\mathbf{u} \cdot \hat{\mathbf{v}} = -\frac{6}{\sqrt{10}} + \frac{5}{\sqrt{10}} = -\frac{1}{\sqrt{10}}
\]

9. \( -\frac{1}{\sqrt{10}} \)

10. An airplane is flying at a bearing of 350° and 415mph and the wind is blowing with a bearing of 50° at 40mph.

a. Find the actual ground speed of the plane.

\[
\mathbf{w} = \left< 40 \cos 40°, 40 \sin 40° \right>
\]

\[
\sqrt{A^2 + B^2} = 436.377\text{mph}
\]

b. Find the direction of the plane relative to the ground.

\[
\tan^{-1} \frac{B}{A} = -84.553°
\]