The semester B examination for Honors Precalculus will consist of two parts. Part 1 is selected response on which a calculator will NOT be allowed. Part 2 is short answer on which a calculator will be allowed.

Items with a symbol next to the item number indicate that a student should be prepared to complete items like these with or without a calculator.

The formulas below will be provided in the examination booklet.

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Compound Interest Formula (continuous): \[ A(t) = Pe^{rt} \]

Compound Interest Formula (n times per year): \[ A(t) = P \left(1 + \frac{r}{n}\right)^{nt} \]

Newton’s Law of Cooling: \[ T(t) = T_M + (T_0 - T_M)e^{-kt} \]

Permutations: \[ _nP_r = \frac{n!}{(n-r)!} \]  
Combinations: \[ _nC_r = \frac{n!}{r!(n-r)!} \]

Parametric Equations for Projectile Motion (distances in feet):

\[ x = (v_o \cos \theta)t \]
\[ y = -16t^2 + (v_o \sin \theta)t + h_o \]

Distance from a Point \((x_1, y_1)\) to the line \(Ax + By + C = 0\):

\[ \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} \]
Vectors:

If \( \vec{u} = \langle u_1, u_2 \rangle \), then

\[
|\vec{u}| = \sqrt{u_1^2 + u_2^2} \text{ with direction angle } \tan^{-1} \left( \frac{u_2}{u_1} \right), \text{ placed in the appropriate quadrant.}
\]

Dot Product of Two Vectors in the Plane \( \vec{u} = \langle u_1, u_2 \rangle \) and \( \vec{v} = \langle v_1, v_2 \rangle \):

\[
\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2
\]

Dot Product of Two Vectors in Space \( \vec{u} = \langle u_1, u_2, u_3 \rangle \) and \( \vec{v} = \langle v_1, v_2, v_3 \rangle \):

\[
\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3
\]

Angle Between Two Vectors \( \vec{u} \) and \( \vec{v} \):

\[
\cos^{-1} \left( \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} \right).
\]

Cross Product of Two Vectors \( \vec{u} = \langle u_1, u_2, u_3 \rangle \) and \( \vec{v} = \langle v_1, v_2, v_3 \rangle \):

\[
\vec{u} \times \vec{v} = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
u_1 & u_2 & u_3 \\
v_1 & v_2 & v_3
\end{vmatrix}
\]

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Law of Cosines: \( c^2 = a^2 + b^2 - 2ab \cos C \)

Law of Sines: \( \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \)

Sum of an Infinite Geometric Series: \( S = \frac{a_1}{1-r} \), \( |r| < 1 \)

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Binomial Theorem:

\[
(a + b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r, \text{where } \binom{n}{r} = \frac{n!}{r!(n-r)!}
\]

OR

\[
(x + y)^n = x^n + \binom{n}{1} x^{n-1} y + \frac{n(n-1)}{1 \cdot 2} x^{n-2} y^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} x^{n-3} y^3 + \ldots + y^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r
\]
Decimal approximations should be given to three places after the decimal point.

For items 1 through 3, sketch the graph of the function. Write the equations of any asymptotes, and the coordinates of any intercepts.

1. \( f(x) = \frac{3}{x^2 - 3x - 10} \)

2. \( f(x) = \frac{x - 4}{x^2 + 2x - 8} \)

3. \( f(x) = 2 + \frac{5}{x - 3} \)

For items 4 and 5, sketch the graph of the function. Write the equations of any asymptotes, state the domain and range, give the \( x \)- and \( y \)-coordinates of any removable discontinuities, and the coordinates of any intercepts.

4. \( f(x) = \frac{x^2 - 7x + 6}{x^2 - 36} \)

5. \( f(x) = \frac{x^2 - 8x + 7}{x - 6} \)

6. Which of the following is the equation of the horizontal asymptote of the graph of the function \( f(x) = \frac{4x^2 - 2}{x^2 - 5} \)?

   A \( x = \frac{2}{5} \)  
   B \( x = 5 \)  
   C \( y = \frac{1}{2} \)  
   D \( y = 4 \)

For items 7 and 8, solve the equations. Check for any extraneous solutions.

7. \( \frac{3x}{x - 1} = \frac{12}{x^2 - 1} + 2 \)

8. \( \frac{x}{x - 2} + \frac{3x}{x - 4} = \frac{32 - 2x}{x^2 - 6x + 8} \)
For items 9 and 10, solve the inequality.

9. \( \frac{x}{x - 2} < 0 \)

10. \( \frac{x - 3}{x^2 + 7x + 10} \geq 0 \)

For items 11 and 12, use partial fraction decomposition to write an expression equivalent to the given expression.

11. \( \frac{13x - 31}{x^2 - 5x + 6} \)

12. \( \frac{-9}{x^2 - x - 2} \)

13. Find the following limit. \( \lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 + 8x - 20} \)

14. Which of the following statements is false?

A. \( f(x) = \ln x \) has a domain of all positive real numbers.

B. \( f(x) = \ln x \) increases without bound as \( x \) increases without bound.

C. The graph of \( g(x) = e^{-x} \) is the graph of \( f(x) = e^x \) reflected about the y-axis.

D. \( g(x) = e^{-x} \) has a range of all real numbers.

15. Which of the following is equivalent to \( \log_5 \left( \frac{5}{x^3} \right) \)?

A. \( 5 - 3 \log_5 x \)

B. \( 1 - 3 \log_5 x \)

C. \( -3 \log_5 x \)

D. \( 2 \log_5 x \)
16. Which of the following is equivalent to \( \log\left(\frac{y^2}{\sqrt{x}}\right) \)?

A  \( 2\log y - 2\log x \)

B  \( 2\log y - \frac{1}{2}\log x \)

C  \( \log(2y) - \log\left(\frac{1}{2}x\right) \)

D  \( \frac{\log(y^2)}{\log(\sqrt{x})} \)

17. Which of the following is equivalent to \( 3\ln x + \ln z - \ln y \)?

A  \( 3\ln \left(\frac{xz}{y}\right) \)

B  \( \ln \left(\frac{xz}{y}\right)^3 \)

C  \( \ln \left(\frac{x^3z}{y}\right) \)

D  \( \ln \left(\frac{(xz)^3}{y}\right) \)

18. Which of the following is equivalent to \( \frac{\log 20}{\log 2} \)?

A  \( 1 \)

B  \( \log 18 \)

C  \( \log_x 20 \)

D  \( \log_{20} 2 \)
For items 19 and 20, evaluate. Write your answer correct to three decimal places.

19. \( \log_7 6904 \)
20. \( \log_4 0.0123 \)

21. The graph of \( g(x) \) is the graph of \( f(x) = 3^x \) reflected about the \( x \)-axis and translated five units to the left. What is the function rule for \( g(x) \)?

22. The graph of \( g(x) \) is the graph of \( f(x) = 9^x \) reflected about the \( y \)-axis and stretched vertically by a factor of 7. What is the function rule for \( g(x) \)?

For items 23 through 30, solve each equation. Your answer may be exact (in terms of logarithms or \( e \)) or correct to three places after the decimal point.

23. \( 3e^{2x} - 7 = 50 \) (exact answer only)
24. \( 2.8^x = 345 \)
25. \( 5 \cdot 2^{3x-1} + 20 = 520 \)
26. \( \ln(x + 4) = 2 \)
27. \( \log_7(3x - 1) = 2 \)
28. \( \log_5 x + \log_5(x - 4) = 1 \)
29. \( \log_2(x + 5) - \log_2(x - 1) = 2 \log_2 2 \)
30. \( 2 - \log x = \log 20 \)

31. A colony of insects has an initial population of 600. The number of insects triples every 4 weeks.
   a. Write a function for the number of insects \( N(t) \) after \( t \) weeks.
   b. What will the number of insects (nearest whole number) be after 7 weeks?
   c. About how many weeks (correct to three decimal places) will the number of insects be 12,000?
32. A population that grows continuously has an initial value of 400. After \( t = 8 \) years, the population is 1,000. What is the continuous growth rate of this population? Your answer should be correct to three places after the decimal point or to the nearest tenth of a percent.

33. Jack puts $5,000 in the bank and earns 6% interest compounded monthly. How many years and months will it take for the amount of money to become $6,000?

34. The population of a certain city \( t \) years after the beginning of 2010 is given by the function \( P(t) = 1300e^{0.04t} \).

   a. What was the population at the beginning of 2010?
   
   b. What is the continuous rate of growth of the population?
   
   c. What will the population be at the beginning of 2020?
   
   d. At what time \( t \) (correct to three decimal places) will the population be 2,500?

35. A full swimming pool has 7,600 gallons in it when it begins leaking at a continuous rate of 7% per hour.

   a. How many gallons of water will be remaining in the pool after 5 hours? Round your answer to the nearest whole number of gallons.
   
   b. At what time \( t \) will the pool be half-full? Round your answer to three places after the decimal point.

36. The percentage of adult height obtained by a boy \( x \) years old is modeled by the function \( f(x) = 29 + 49 \log(x+1) \).

   a. Approximately what percentage of his adult height is a nine year old?
   
   b. At what age, correct to three decimal places) will a boy reach 90 percent of his adult height?
37. For the function \( g(x) = -\ln(x + 2) - 5 \),
   a. What are the domain and range?
   b. What are the equations of any asymptotes?
   c. Describe the transformations of the graph of \( f(x) = \ln x \) that results in the graph of \( g(x) \)?

38. The logistic function \( f(t) = \frac{30000}{1 + 19e^{-1.5t}} \) describes the number of people \( f(t) \) that have become ill will the flu \( t \) weeks after its initial outbreak.
   a. How many people became ill when the outbreak first began?
   b. After approximately how many weeks were 10,000 people ill?
   c. What is the maximum number of people that can become ill?

For items 39 through 41, eliminate the parameter to find the relationship between \( x \) and \( y \).

39. \[
\begin{align*}
  x &= 3t \\
  y &= 6t - 7
\end{align*}
\]

40. \[
\begin{align*}
  x &= 2t + 4 \\
  y &= 4t + 5
\end{align*}
\]

41. \[
\begin{align*}
  x &= 5\cos t \\
  y &= 8\sin t
\end{align*}
\]

42. A golfer hits a ball from the ground at a 20° angle to the horizontal with an initial velocity of 132 feet per second.
   a. Model the motion of the ball with a set of parametric equations for the horizontal and vertical distances that the ball is from the golfer after \( t \) seconds.
   b. Give the location of the ball after 2 seconds (correct to three places after the decimal point).
   c. How far is the ball from the golfer when it hits the ground? Give your answer correct to three places after the decimal point.
43. A punter kicks a football from an initial height of 2 feet with an initial velocity of 82 feet per second at an angle of 63° with the horizontal.
   a. Write parametric equations for the horizontal and vertical distances that the ball is from the punter after \( t \) seconds.
   b. What is the position of the ball at \( t = 1.5 \) seconds?
   c. A player 6 feet tall is standing 50 yards (150 feet) from the punter directly in the path of the kick. Will the ball go over his head? Justify your answer.
   d. Suppose that the ball hits the ground. When will the ball hit the ground and how far from the punter will the ball hit the ground? Give both answers correct to three decimal places.

44. Write vector and parametric equations of the line that passes through the points \((1, 4)\) and \((3, 9)\).

45. Given the points \(A(2, -1)\) and \(B(0, 4)\), what is \(\overrightarrow{AB}\) in component form?
   
   A \(\langle2, -5\rangle\)
   
   B \(\langle-2, 5\rangle\)
   
   C \(\langle2, 3\rangle\)
   
   D \(\langle-2, -3\rangle\)

46. What is the magnitude and direction (to the nearest tenth of a degree) of the vector \(\langle-6, 8\rangle\)?

Look at the vectors \(\vec{u}\) and \(\vec{v}\) below.

47. Represent graphically the vector \(\vec{u} + \vec{v}\) using the head-to-tail and parallelogram methods.

48. Represent the vector \(\vec{u} - \vec{v}\) graphically.
For items 49 through 52, use the vectors \( \vec{a} = \langle 2, -3 \rangle, \vec{b} = \langle 6, 5 \rangle, \) and \( \vec{c} = \langle r, -9 \rangle \).

49. \( \vec{a} \cdot \vec{b} = \)

50. Find the measure of the angle between \( \vec{a} \) and \( \vec{b} \) to the nearest tenth of a degree.

51. Find the value of \( r \) such that the vectors \( \vec{a} \) and \( \vec{c} \) are parallel.

52. Find the value of \( r \) such that the vectors \( \vec{b} \) and \( \vec{c} \) are perpendicular.

53. A vector in space has initial point \( A(-2, -3, 4) \) and terminal point \( B(-1, 4, -5) \).
   a. Write \( \overrightarrow{AB} \) in component form.
   b. What is the magnitude of \( \overrightarrow{AB} \).

54. Let \( \vec{u} = \langle -4, 1, 5 \rangle \) and \( \vec{v} = \langle -3, 2, -1 \rangle \) represent two vectors in space.
   a. Determine the angle between the vectors to the nearest tenth of a degree.
   b. Determine the cross product of the vectors and state the geometric meaning of your answer.
   c. Write the vector equation of the line that is parallel to \( \vec{u} \) and passes through the point \( P(2, 4, -3) \).

55. A pilot sets his plane to travel due west at 400 miles per hour. A wind is blowing towards the northwest at 60 miles per hour that blows the plane off course.
   a. What is the ground speed of the plane, correct to three decimal places?
   b. In which direction is the plane travelling, correct to three decimal places?

For items 56 and 57, write the equivalent rectangular coordinates for each point in polar form.

56. \( \left( 8, \frac{\pi}{3} \right) \)

57. \( \left( -4, -\frac{7\pi}{4} \right) \)
For items 58 and 59, write the equivalent polar form for each point in rectangular form.

58. \((4, -\sqrt{3})\)

59. \((-1, -1)\)

60. Which of the following represents the same point as \(4, -\frac{\pi}{3}\)?

A \(\left(4, \frac{\pi}{3}\right)\)

B \(\left(-4, \frac{2\pi}{3}\right)\)

C \(\left(4, \frac{2\pi}{3}\right)\)

D \(\left(-4, \frac{5\pi}{3}\right)\)

For items 61 and 62, graph each polar equation.

61. \(r = 1 + 2 \cos \theta\)

62. \(r = 4 \sin (2\theta)\)

For items 63 and 64, write each rectangular equation in polar form.

63. \(x + y = 9\)

64. \(x = 7\)

For items 65 and 66, write each polar equation in rectangular form.

65. \(r \sin \theta = 6\)

66. \(r = \frac{3}{2 \cos \theta + 5 \sin \theta}\)
For items 67 and 68, write the complex number in polar form.

67. \(-6\sqrt{3} + 6i\)

68. \(3 + i\sqrt{3}\)

For items 69 and 70, write the complex number in rectangular form.

69. \(6\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)\)

70. \(-2\left(\cos7\pi + i\sin7\pi\right)\)

For items 71 and 72, simplify. Write your answers in polar form.

71. \[
\frac{\left[3\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right] \cdot 4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)}{6\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)}
\]

72. \[
\left[3\left(\cos55^\circ + i\sin55^\circ\right)\right]^4
\]

For items 73 and 74, determine the coordinates \((r, \theta)\) that are solutions to the system.

73. \[
\begin{align*}
    r &= 2\cos\theta \\
    r &= 1 - 3\cos^2\theta
\end{align*}
\]
where \(0^\circ \leq \theta < 360^\circ\), with \(\theta\) to the nearest tenth of a degree

74. \[
\begin{align*}
    r &= \sin\theta \\
    r &= \sin(2\theta)
\end{align*}
\]
where \(0 \leq \theta < 2\pi\)

For items 75 and 76, represent the series using summation notation.

75. \(8 + 16 + 32 + 64 + \ldots\)

76. \(11 + 15 + 19 + 23 + 27 + 31 + 35\)
For items 77 through 79, determine the sum, if any, for the infinite series.

77. \[9 + 3 + \frac{1}{3} + \ldots\]

78. \[\sum_{n=1}^{\infty} 3 \cdot 2^n\]

79. \[\sum_{n=1}^{\infty} 8 \left(\frac{3}{4}\right)^n\]

For items 80 and 81, evaluate.

80. \[\sum_{n=1}^{4} (3n + 1)\]

81. \[\sum_{n=5}^{10} (4n)\]

82. Given that \(|x| > 1\), in terms of \(x\), what is the sum of the series \(1 + x + x^2 + x^3 + x^4 + \ldots\)?

83. Expand. \((x - 2)^4\)

84. What is the coefficient of \(x^3\) in the expansion of \((x + 2)^5\)?

85. What is the coefficient of the \(x^3y\) term in the expansion of \((x^3 + 5y)^4\)?

86. What is the distance from the point \((2, -3)\) and the line whose equation is \(3x - 4y + 12 = 0\)?